

Theory of Optical Bloch Oscillations in the Zigzag Waveguide Array

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Abstract

The Bloch oscillations in the zigzag array of the optical waveguides are considered. The multiple scattering formalism (MSF) is used for the numerical simulation of the optical beam which propagates within the array. The effect of the second-order coupling which depends on the geometrical parameters of the array is investigated. The results obtained within the MSF are compared with the calculation based on the phenomenological coupling modes model. The calculations are performed for the waveguides fabricated in alkaline earth boro-aluminosilicate glass sample, which are the most promising for the C-band ($\lambda \sim 1530 - 1565$ nm).

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I. INTRODUCTION

The rapid development of technologies for production of optical waveguides provides a possibility of optical simulation of quantum phenomena inherent in the condensed-matter physics [1], such as Bloch oscillations [2–7], Zenner tunneling [8–10], Anderson localization [11, 12], dynamic localization [13, 14]. Periodic arrays of optical waveguides are a special case of photonic crystals. Such systems are of interest due to the band structure of their optical spectrum that is similar to the band structure in the electronic spectrum of ordinary crystals [15, 16]. Because of this, the behavior of light in photonic crystals is analogous to the behavior of electron wave function in a periodic potential of ordinary crystals. This fact is of considerable interest for practical applications because it allows to steer the optical signal effectively. Besides, it is interesting from a purely scientific point of view, since the optical counterparts of quantum phenomena can be observed directly.

Usually, the theoretical model of the light propagation in the optical waveguides array takes into account only the next neighbors coupling (NNC). This model is applicable for the particular case of the plane arrays, since the high order coupling is too weak. But sometimes the second-order coupling (SOC) should be taken into account. The most evident example of a system with the significant SOC is the zigzag array (see Fig. 1). For such a system the SOC depends on the angle θ . Varying this angle, we can make the SOC comparable or even greater than the NNC. The SOC can influence the diffraction processes [17] and forming of the solitons in the case of the nonlinear optics [18, 19].

In this paper we investigate the influence of SOC on the optical Bloch oscillations. The phenomenon of Bloch oscillations (BO) takes place in the array with the monotonic change of the refractive index of the waveguides as one passes from one homogeneous waveguide to another. For the case of the plane array, where only the NNC should be taken into account, the light beam path possesses an oscillatory form. This effect is widely investigated both experimentally and theoretically in [2–6]. But for the zigzag array, where the SOC is considerable, the path of the optical beam takes more complex form. This phenomenon, known as the anharmonic Bloch oscillation (ABO), was predicted in [20] and experimentally confirmed in [21].

Numerical simulation of various phenomena in systems of interacting optical waveguides, including BO and ABO, is usually based on a coupled modes model. The parameters of this model are typically determined experimentally.

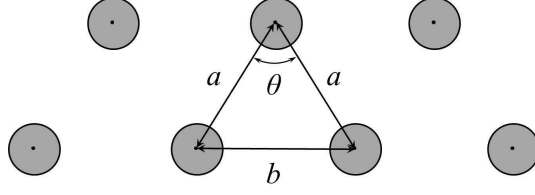


Figure 1: Zigzag optical waveguide array.

In our works [22, 23] we proposed another method of numerical simulation which requires no additional data except of the geometrical and optical parameters of the array (radii of the waveguides and distances between them, the refractive indices of the waveguides and environment). Our method uses the multiple scattering formalism based on the macroscopic electrodynamics approach. In papers [22, 23] we presented the general algorithm for calculating the path of the optical beam in an array of waveguides.

In the present work we generalize the results obtained in [22, 23]. We apply the multiple scattering formalism to the ABO calculation in the zigzag array. The obtained picture of ABO is compared with the results of calculation with use of the coupling mode model. For this purpose, we derive the analytical formula for the path of the optical beam in the zigzag array, using the coupling mode model with the SOC taken into account. Besides, we use the MSF to obtain the dependence of the coupling constant of the SOC as function of the angle θ (see Fig. 1).

Recently a new method to fabricate low bend loss femtosecond-laser written waveguides was developed [24, 25]. The waveguides are fabricated in alkaline earth boro-aluminosilicate glass sample, so they are the most effective for the C-band ($\lambda \sim 1530 - 1565 \text{ nm}$). In this work we consider a sample that can be fabricated by means of this technology.

This paper is organized as follows. In Sec. II we describe the multiple scattering formalism. In subsection II A we present the algorithm for calculating the path of the optical beam in an array of waveguides. In subsection II B we derive the equation of the coupling modes model from the MSF and present the formulae for the coupling constants. In Sec. III we represent the pictures of ABO obtained by use of MSF for different θ . We compare these results with the paths of the optical beam described by the coupling modes model. Besides, in Sec. III we present the dependence of the SOC coupling constant on the angle θ .

II. MULTIPLE SCATTERING FORMALISM

Let us consider the array of N parallel infinite cylindrical dielectric waveguides. The axes of the waveguides are parallel to the z -axis. The distance between the axes of the adjacent waveguides is denoted by a . All the waveguides are assumed to possess the same radius R but different refractive indices n_j , j being the waveguide number. The refractive index of the environment is n^0 . The permeability of the waveguide material and the environment is unity.

Suppose that a guided mode with a frequency ω is excited within the array. Then, all the components of the electromagnetic field are proportional to $e^{-i\omega t + i\beta z}$.

Let us consider the field of the guided mode inside of the array. The field of the guided mode is finite in any point into the waveguide. So, the electromagnetic field inside of the j -th waveguide may be represented in the form

$$\begin{aligned}\tilde{\mathbf{E}}_j(\mathbf{r}) &= e^{-i\omega t + i\beta z} \sum_{m=0, \pm 1, \dots} e^{im\phi_j} \left(c_{jm} \tilde{\mathbf{N}}_{\omega_j \beta m}(\rho_j) - d_{jm} \tilde{\mathbf{M}}_{\omega_j \beta m}(\rho_j) \right), \\ \tilde{\mathbf{H}}_j(\mathbf{r}) &= e^{-i\omega t + i\beta z} n_j \sum_{m=0, \pm 1, \dots} e^{im\phi_j} \left(c_{jm} \tilde{\mathbf{M}}_{\omega_j \beta m}(\rho_j) + d_{jm} \tilde{\mathbf{N}}_{\omega_j \beta m}(\rho_j) \right), \quad \rho_j < R.\end{aligned}\tag{1}$$

Here $\omega_j = n_j \omega$, and ρ_j, ϕ_j are the cylindrical coordinates of the vector $\mathbf{r} - \mathbf{r}_j$, where \mathbf{r}_j is the coordinates of the axis of the j -th waveguide. The vector cylinder harmonics $\tilde{\mathbf{M}}_{\omega_j \beta m}(\rho_j)$ and $\tilde{\mathbf{N}}_{\omega_j \beta m}(\rho_j)$ are defined as follows

$$\tilde{\mathbf{N}}_{\omega_j \beta m}(\rho_j) = \mathbf{e}_r \frac{i\beta}{\varkappa_j} J'_m(\varkappa_j \rho_j) - \mathbf{e}_\phi \frac{m\beta}{\varkappa_j^2 \rho_j} J_m(\varkappa_j \rho_j) + \mathbf{e}_z J_m(\varkappa_j \rho_j),\tag{2}$$

$$\tilde{\mathbf{M}}_{\omega_j \beta m}(\rho_j) = \mathbf{e}_r \frac{m\omega_j}{\varkappa_j^2 \rho_j} J_m(\varkappa_j \rho_j) + \mathbf{e}_\phi \frac{i\omega_j}{\varkappa_j} J'_m(\varkappa_j \rho_j),\tag{3}$$

where $\varkappa_j = \sqrt{\omega_j^2 - \beta^2}$, $J_m(\varkappa_j \rho_j)$ is the Bessel function, and the prime means the derivative with respect to the argument $\varkappa_j \rho_j$.

Let us turn to the electromagnetic field outside of the array. This field is the sum of the contributions of all the waveguides,

$$\mathbf{E}(\mathbf{r}) = \sum_{j=1}^N \mathbf{E}_j(\mathbf{r}), \quad \mathbf{H}(\mathbf{r}) = \sum_{j=1}^N \mathbf{H}_j(\mathbf{r}).\tag{4}$$

The contribution induced by the j -th waveguide and vanishing at $\rho_j \rightarrow \infty$ may be represented in

the form

$$\begin{aligned}\mathbf{E}_j(\mathbf{r}) &= e^{-i\omega t + i\beta z} \sum_m e^{im\phi_j} \left(a_{jm} \mathbf{N}_{\omega'\beta m}(\rho_j) - b_{jm} \mathbf{M}_{\omega'\beta m}(\rho_j) \right), \\ \mathbf{H}_j(\mathbf{r}) &= e^{-i\omega t + i\beta z} n^0 \sum_m e^{im\phi_j} \left(a_{jm} \mathbf{M}_{\omega'\beta m}(\rho_j) + b_{jm} \mathbf{N}_{\omega'\beta m}(\rho_j) \right), \quad \rho_j > R.\end{aligned}\tag{5}$$

Here $\omega' = n^0\omega$. The vector cylinder harmonics $\tilde{\mathbf{M}}_{\omega_j\beta m}(\rho_j)$ and $\tilde{\mathbf{N}}_{\omega_j\beta m}(\rho_j)$ are

$$\mathbf{N}_{\omega'\beta m}(\rho_j) = \mathbf{e}_r \frac{i\beta}{\kappa'} H'_m(\kappa' \rho_j) - \mathbf{e}_\phi \frac{m\beta}{\kappa'^2 \rho_j} H_m(\kappa' \rho_j) + \mathbf{e}_z H_m(\kappa' \rho_j),\tag{6}$$

$$\mathbf{M}_{\omega'\beta m}(\rho_j) = \mathbf{e}_r \frac{m\omega'}{\kappa'^2 \rho_j} H_m(\kappa' \rho_j) + \mathbf{e}_\phi \frac{i\omega'}{\kappa'} H'_m(\kappa' \rho_j),\tag{7}$$

where $\kappa' = \sqrt{\omega'^2 - \beta^2}$, and $H_m(\kappa' \rho_j)$ is the Hankel function of the first kind. Note that, for $\beta = 0$, Eqs. (1) and (5) transform into the corresponding expressions in Ref. [24], however different notations are used there.

Below we consider the simplest approximation to these equations, namely, the *zero-harmonic approximation*. This means that in Eqs. (1) and (5) only the terms with $m = 0$ are taken into account. In this approximation there are two kinds of the guided modes, namely the transverse magnetic (TM) and transverse electric (TE) modes. For the TM-mode $b_{j0} = d_{j0} = 0$, and for the TE-mode $a_{j0} = c_{j0} = 0$. As an example, let us consider the TM-modes.

$$\tilde{\mathbf{E}}_j(\mathbf{r}) = c_j \tilde{\mathbf{N}}_{\omega_j\beta 0}(\rho_j), \quad \tilde{\mathbf{H}}_j(\mathbf{r}) = c_j n_j \tilde{\mathbf{M}}_{\omega_j\beta 0}(\rho_j) \quad \rho_j < R.\tag{8}$$

$$\mathbf{E}_j(\mathbf{r}) = a_j \mathbf{N}_{\omega'\beta 0}(\rho_j), \quad \mathbf{H}_j(\mathbf{r}) = a_j n^0 \mathbf{M}_{\omega'\beta 0}(\rho_j) \quad \rho_j > R.\tag{9}$$

Here and below, a_j and c_j stand for a_{j0} and c_{j0} , and the factor $e^{-i\omega t + i\beta z}$ is omitted.

To derive the equations that determine the partial amplitudes a_j and c_j , one should use the boundary conditions on the surface of every waveguide. The boundary conditions on the surface of the j -th waveguide connect the field $\tilde{\mathbf{E}}_j(\mathbf{r})$, $\tilde{\mathbf{H}}_j(\mathbf{r})$ inside the j -th waveguide and the field $\mathbf{E}(\mathbf{r})$, $\mathbf{H}(\mathbf{r})$ outside. In general, there are four independent boundary conditions [23]. However, for $m = 0$ TM-modes and only two boundary conditions are required:

$$[\mathbf{E}(\mathbf{R}_j)]_z = [\tilde{\mathbf{E}}_j(\mathbf{R}_j)]_z, \quad [\mathbf{H}(\mathbf{R}_j)]_\phi = [\tilde{\mathbf{H}}_j(\mathbf{R}_j)]_\phi,\tag{10}$$

here \mathbf{R}_j is the radius-vector of a point on the surface of the j -th waveguide.

The boundary conditions on the surface of the j -th waveguide can be expressed in the most convenient form if the contributions of all the waveguides to the field outside the array are expressed

in terms of the same argument ρ_j . For this propose we apply the following relations:

$$\begin{aligned}\mathbf{N}_{\omega'\beta 0}(\rho_l) &\approx U_{lj}(\omega, \beta) \tilde{\mathbf{N}}_{\omega'\beta 0}(\rho_j), \\ \mathbf{M}_{\omega'\beta 0}(\rho_l) &\approx U_{lj}(\omega, \beta) \tilde{\mathbf{M}}_{\omega'\beta 0}(\rho_j), \quad l \neq j,\end{aligned}\tag{11}$$

where $U_{lj}(\omega, \beta) = H_0(\varkappa' r_{lj})$, H_0 being the Hankel function of the first kind and r_{lj} being the distance between the axes of the j -th and the l -th waveguides. The relations (11) follow from the Graf theorem (see [25]) in the zero-harmonic approximation.

Thus, it follows from (9) that

$$\mathbf{E}_l(\mathbf{r}) = a_j U_{lj}(\omega, \beta) \tilde{\mathbf{N}}_{\omega'\beta 0}(\rho_j), \quad \mathbf{H}_l(\mathbf{r}) = a_j n^0 U_{lj}(\omega, \beta) \tilde{\mathbf{M}}_{\omega'\beta 0}(\rho_j).\tag{12}$$

Substituting (12) to (4), one gets

$$\begin{aligned}\mathbf{E}(\mathbf{r}) &= a_j \mathbf{N}_{\omega'\beta 0}(\rho_j) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) \tilde{\mathbf{N}}_{\omega'\beta 0}(\rho_j), \\ \mathbf{H}(\mathbf{r}) &= a_j n^0 \mathbf{M}_{\omega'\beta 0}(\rho_j) + \sum_{l \neq j} a_l n^0 U_{lj}(\omega, \beta) \tilde{\mathbf{M}}_{\omega'\beta 0}(\rho_j).\end{aligned}\tag{13}$$

So, the boundary conditions (10) take the form

$$\begin{aligned}a_j H_0(\varkappa' R) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) J_0(\varkappa' R) &= c_j J_0(\varkappa_j R), \\ a_j i \frac{n^0 \omega'}{\varkappa'} H'_0(\varkappa' R) + \sum_{l \neq j} a_l U_{lj}(\omega, \beta) i \frac{n^0 \omega'}{\varkappa'} J'_0(\varkappa' R) &= c_j i \frac{n_j \omega_j}{\varkappa_j} J'_0(\varkappa_j R).\end{aligned}\tag{14}$$

Eqs. (14) lead to the following system of equations:

$$\frac{a_j}{\bar{a}_j(\omega, \beta)} - \sum_{l \neq j} U_{jl}(\omega, \beta) a_l = 0,\tag{15}$$

$$c_j = \bar{c}_j(\omega, \beta) a_j.\tag{16}$$

Here

$$\bar{a}_j(\omega, \beta) = \frac{\varepsilon_j \varkappa' J'_0(\varkappa_j R) J_0(\varkappa' R) - \varepsilon^0 \varkappa_j J_0(\varkappa_j R) J'_0(\varkappa' R)}{\varepsilon^0 \varkappa_j J_0(\varkappa_j R) H'_0(\varkappa' R) - \varepsilon_j \varkappa' J'_0(\varkappa_j R) H_0(\varkappa' R)},\tag{17}$$

$$\bar{c}_j(\omega, \beta) = \frac{\varepsilon^0 \varkappa_j \{H_0(\varkappa' R) J'_0(\varkappa' R) - H'_0(\varkappa' R) J_0(\varkappa' R)\}}{\varepsilon^0 \varkappa_j J'_0(\varkappa' R) J_0(\varkappa_j R) - \varepsilon_j \varkappa' J_0(\varkappa' R) J'_0(\varkappa_j R)}.\tag{18}$$

A. Method for the optical beam trajectory calculation.

The system of equations (15) describes the guided modes of the array of waveguides. This system possesses the nontrivial solution only if the determinant of the matrix of this system vanishes,

$$\det \left\| \frac{\delta_{jl}}{\bar{a}_j(\omega, \beta)} - U_{jl}(\omega, \beta) \right\| = 0. \quad (19)$$

This equation allows to obtain the propagation constants β_n of the guided modes for the given frequency ω , n being the number of a guided mode. There are N solutions of Eq. (19).

Let $a_j(\beta_n)$ be the normalized solution of Eq. (15), $\sum_{j=1}^N |a_j(\beta_n)|^2 = 1$. The guided mode of the frequency ω is a superposition of the modes with different β_n :

$$\begin{aligned} \mathbf{E}(t, \mathbf{r}) &= e^{-i\omega t} \sum_n C_n e^{i\beta_n z} \sum_{j=1}^N a_j(\beta_n) \mathbf{N}_{\omega' \beta_n 0}(\rho_j), \\ \mathbf{H}(t, \mathbf{r}) &= n^0 e^{-i\omega t} \sum_n C_n e^{i\beta_n z} \sum_{j=1}^N a_j(\beta_n) \mathbf{M}_{\omega' \beta_n 0}(\rho_j). \end{aligned} \quad (20)$$

The coefficients C_n determine the superposition.

Let us introduce the modal amplitude

$$a_j(z) = \sum_n C_n e^{i\beta_n z} a_j(\beta_n). \quad (21)$$

The functions $\mathbf{N}_{\omega' \beta_n 0}(\rho_j)$, $\mathbf{M}_{\omega' \beta_n 0}(\rho_j)$ vanish rapidly as ρ_j increases. So, the field near the j -th waveguide is mainly determined by the partial amplitudes $a_j(\beta_n)$. So, the modal amplitude $a_j(z)$ represents the behavior of the guided modes properly. The coefficients C_n are obtained from the boundary condition at $z = 0$:

$$\sum_n C_n a_j(\beta_n) = a_j(0). \quad (22)$$

The system of equations (22) allows to obtain the coefficients C_n for given $a_j(0)$.

Below we suppose that the boundary conditions $a_j(0)$ possess the form of the Gaussian beam,

$$a_j(0) = e^{-\frac{(j-j_0)^2}{\sigma^2} + ik_0 a j}. \quad (23)$$

This means that the external source approximately illuminates the ends of the waveguides with the numbers $j_0 - \sigma < j < j_0 + \sigma$ and the phase difference between the amplitudes taken at the ends of the nearest waveguides is $k_0 a$.

Thus, the guided mode can be found as follows:

- 1) Calculate numerically the set of propagating constants β_n using Eq. (19);
- 2) Obtain the amplitudes $a_j(\beta_n)$ for every β_n using Eq. (15);
- 3) For the given boundary conditions find the coefficients C_n using Eq. (22);
- 4) Calculate the function $a_j(z)$ by means of Eq. (21).

B. Derivation of the coupled modes equation

Suppose that the refractive index is $n_j = n_0 + \delta n \times j$, $\delta n \ll n_0$. Then, the solutions $\beta_j^{(0)}$ of the equation

$$\frac{1}{\bar{a}_j(\omega, \beta_j^{(0)})} = 0 \quad (24)$$

may be represented in the following form:

$$\beta_j^{(0)} = \beta_0^{(0)} + \alpha j, \quad \alpha \ll \beta_0^{(0)}. \quad (25)$$

For the case of the weak coupling, $\left| (\beta_n - \beta_j^{(0)}) / \beta_j^{(0)} \right| \ll 1$. Then, one has

$$\frac{1}{\bar{a}_j(\beta_n)} \approx \frac{\partial}{\partial \beta} \frac{1}{\bar{a}_j(\beta)} \Big|_{\beta=\beta_j^{(0)}} \times (\beta_n - \beta_j^{(0)}). \quad (26)$$

Then, Eq. (15) leads to the following result:

$$(\beta_n - \beta_j^{(0)}) a_j(\beta_n) - \sum_{l \neq j} \gamma_{lj} a_l(\beta_n) = 0. \quad (27)$$

where

$$\gamma_{lj}(\beta_n) = \frac{U_{lj}(\omega, \beta_n)}{\frac{\partial}{\partial \beta} \frac{1}{\bar{a}_j(\beta)} \Big|_{\beta=\beta_j^{(0)}}}. \quad (28)$$

For the zigzag array we can take into account the first-order and second-order coupling only, i.e. $l = j \pm 1$ and $l = j \pm 2$. Due to the symmetry, $\gamma_{j,j-1} = \gamma_{j,j+1}$ and $\gamma_{j,j-2} = \gamma_{j,j+2}$. Besides, since the waveguides differ insignificantly, one can neglect the dependence of constants $\gamma_{j,j\pm 1}$ and $\gamma_{j,j\pm 2}$ on j . Then, for the case of weak coupling and insignificant difference of the waveguides one can neglect the dependence of $\gamma_{lj}(\beta_n)$ on β_n . Thus, Eq. (27) takes the form

$$(\beta_n - \beta_j^{(0)}) a_j(\beta_n) - \gamma_1 (a_{j-1}(\beta_n) + a_{j+1}(\beta_n)) - \gamma_2 (a_{j-2}(\beta_n) + a_{j+2}(\beta_n)) = 0. \quad (29)$$

Here γ_1 and γ_2 stand for $\gamma_{j,j\pm 1}$ and $\gamma_{j,j\pm 2}$ correspondingly.

Now we can apply the obtained Eq. (29) to formulate the equation for the function $a_j(z)$. It follows from (29) that the function $a_j(z)$ defined by Eq. (21) with any set of coefficients C_n satisfies the equation of the coupled modes.

$$\left(i\frac{d}{dz} + \beta_j^{(0)}\right) a_j(z) + \gamma_1 \left(a_{j-1}(z) + a_{j+1}(z)\right) + \gamma_2 \left(a_{j-2}(z) + a_{j+2}(z)\right) = 0. \quad (30)$$

III. PATH OF THE OPTICAL BEAM

We apply the developed technique for calculating the optical beam in an array represented in Fig. 1. The parameters taken for the calculation correspond approximately to the parameters of the arrays of waveguides reported in work [26].

The wavelength of the laser source in [26] is $\lambda = 1550$ nm. We take the waveguide radius $R = 5\lambda = 7750$ nm. The refractive index of the environment is $n^0 = 1.4877$, and the refractive index of the waveguide $j = 0$ (the center of the array) is $n_0 = n^0 + 5 \times 10^{-3}$. These parameters also approximately corresponds to the experiments reported in [26]. For our calculations we assume that the separation between the waveguides is $a = 3R$, and a variation of refractive indices between the nearest waveguides is $\delta n = n_j - n_{j-1} = 5 \times 10^{-6}$. We take the boundary conditions in the form (23) with $\sigma = 4$ and $k_0 = 0$.

We produce the calculation for different values of the angle $\theta = 50^\circ, 60^\circ, 70^\circ, 180^\circ$ (the last case coincides with the simple plane array). The results of the calculations are represented in Figs. 2 – 5.

Every of these figures contains a dotted curve representing the trajectory calculated by means of the coupled modes model. The parameter α is found with the help of Eq. (24). The coupling constants γ_1 and γ_2 for all the values θ are calculated by the use of Eq. (28). The values α and γ_1 do not depend on θ and they are the same for all the four Figs. 2 – 5: $\alpha = 16.48 \text{ m}^{-1}$, $\gamma_1 = -58.44 \text{ m}^{-1}$. But γ_2 depends on the distance b between the second neighbors, so γ_2 decreases with increasing θ : $\gamma_2(50^\circ) = -196.63 \text{ m}^{-1}$, $\gamma_2(60^\circ) = -58.44 \text{ m}^{-1}$, $\gamma_2(70^\circ) = -18.64 \text{ m}^{-1}$, $\gamma_2(180^\circ) = -0.0277 \text{ m}^{-1}$. The dependence of the relation γ_2/γ_1 on the angle θ is depicted in Fig. 6.

Note that for $\theta = 60^\circ$ one has $\gamma_2 = \gamma_1$. This is quite expected result, since for this case $b = a$. Finally, note that for $\theta = 180^\circ$ one has $\gamma_2/\gamma_1 \sim 10^{-4}$. Thus, we confirm that for the plane array of waveguides one can neglect the SOC.

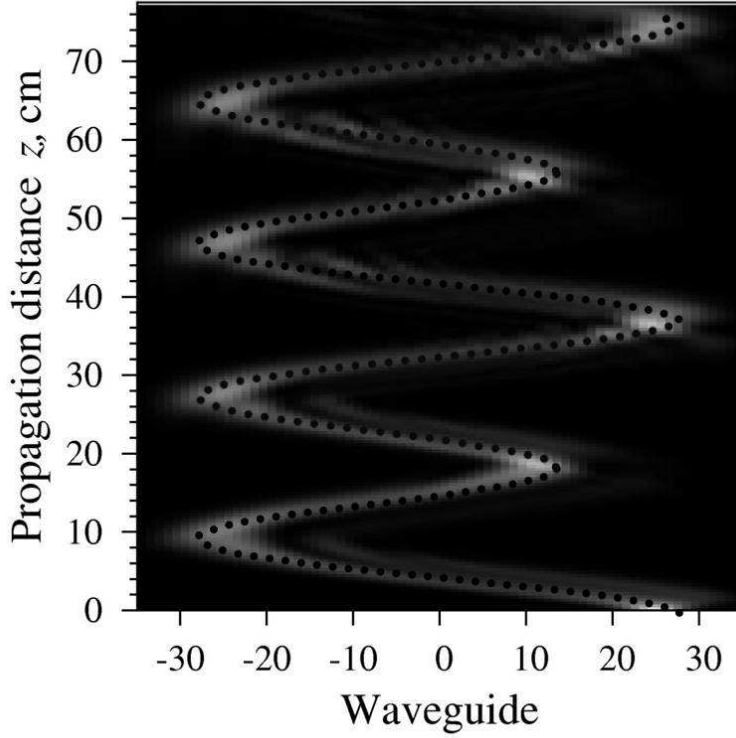


Figure 2: $\theta = 50^\circ$

IV. CONCLUSION

In this paper we investigated the anharmonic Bloch oscillations in the zigzag array of the optical waveguides. For this purpose we applied the multiple scattering formalism based on the macroscopic electrodynamics approach. For the simplicity, we used the zero-harmonic approximation taking into account the TM-modes only. On the basis of the MSF, we developed the numerical algorithm for calculating the optical beam path for the specified boundary conditions at $z = 0$. We chose the boundary conditions possessing the form of the Gaussian wave packet. It was shown that for this case the optical beam path takes the complicated periodic form.

This result is compared with the optical beam path predicted by means of the coupled modes model with the second-order coupling taken into account. This model contains some parameters, namely, the difference between the propagation constants of the adjacent waveguides $\alpha = \beta_j^{(0)} - \beta_{j-1}^{(0)}$ and the coupling constants γ_1 and γ_2 for the first-order and the second-order coupling. These parameters were calculated by means of MSF. We showed that the results of calculations by two different methods are exactly the same.

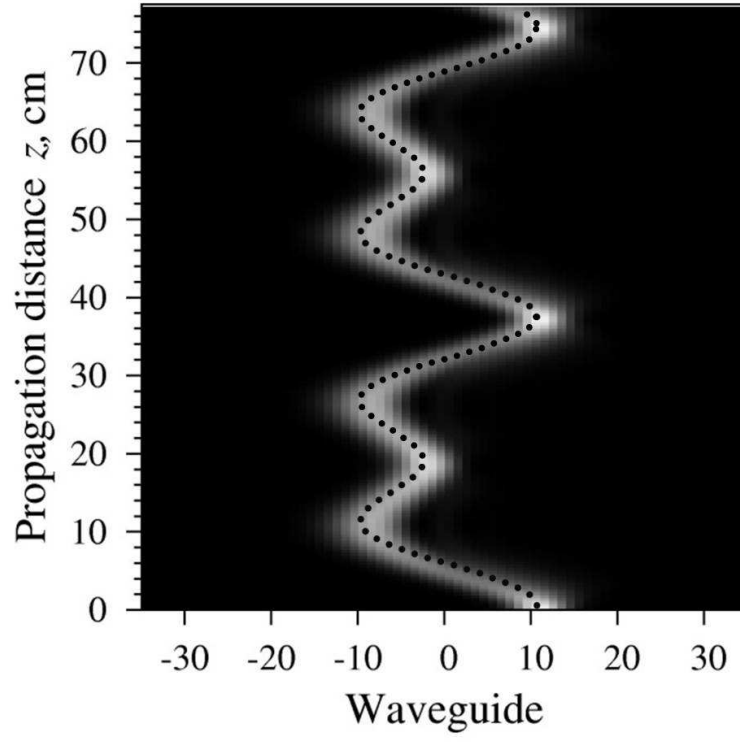


Figure 3: $\theta = 60^\circ$

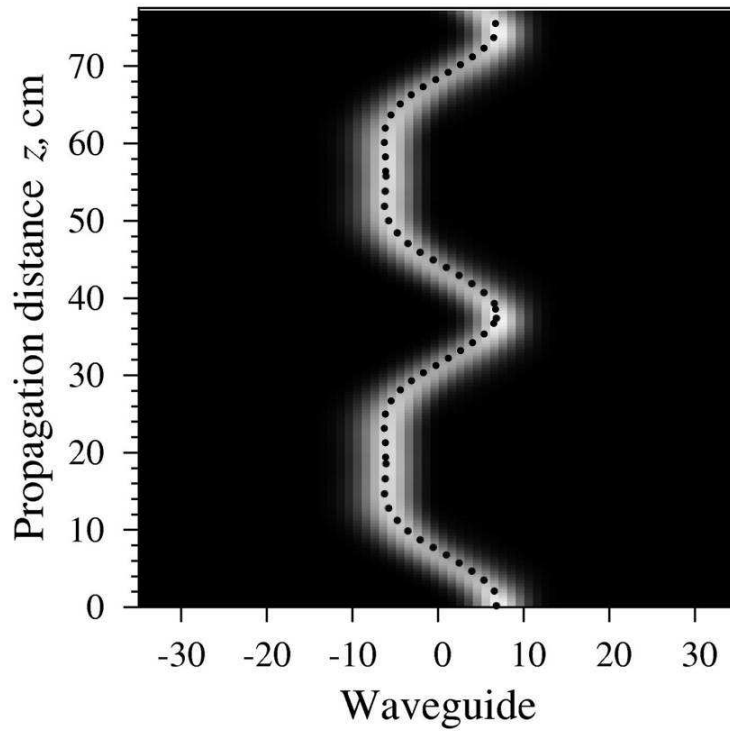


Figure 4: $\theta = 70^\circ$

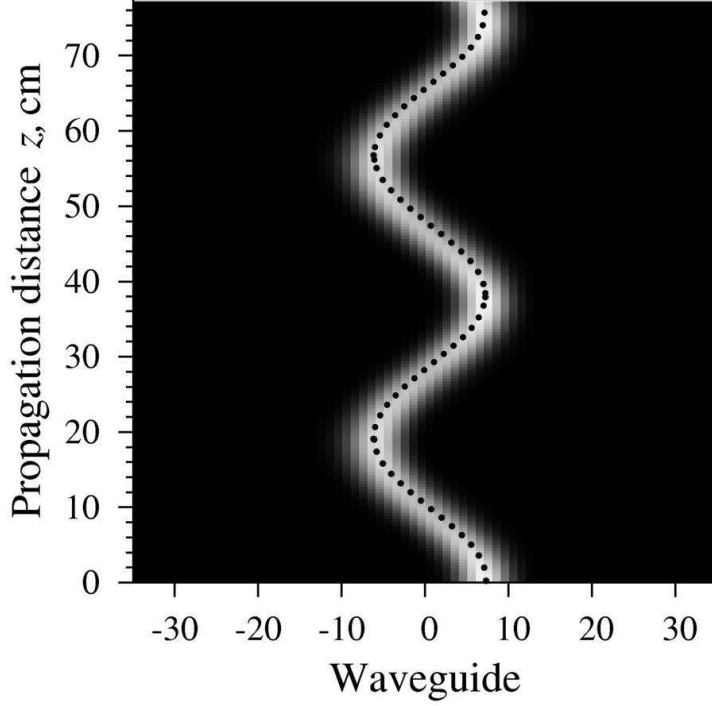


Figure 5: $\theta = 180^\circ$

Besides, we investigated the dependence of the coupling constant of the second-order coupling γ_2 on the angle θ . It was shown that γ_2 decreases quickly as the angle θ increases. For $\theta = 180^\circ$ the value γ_2 becomes negligible. This result confirms that for the plane array it is possible to produce the calculations taking into account the next neighbors coupling only.

Acknowledgments

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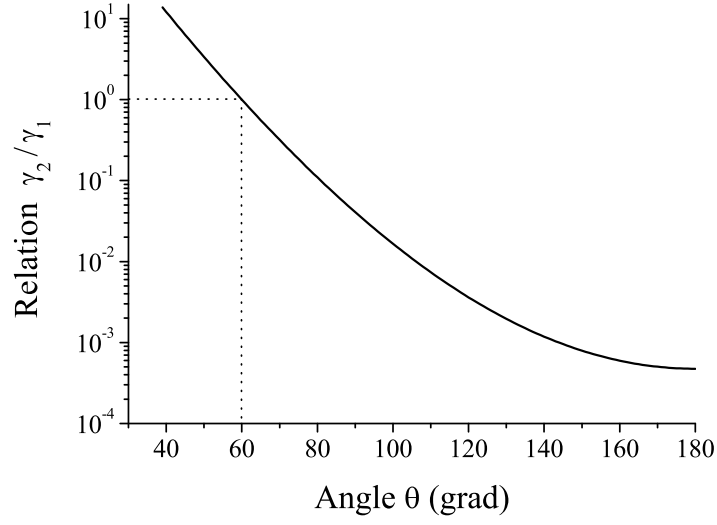


Figure 6: The dependence of the relation γ_2/γ_1 on the angle θ .

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